CS 311: Discrete Structures
Fall 1992

Handout 1: Logarithms and exponentiation

Definition: For b a real number and n a natural number called the *exponent*, b^n is defined as the product of b with itself n times:

$b^n = b \cdot b \cdot b \cdots b \quad (n \text{ times})$

For $n = 0$, $b^0 = 1$.

This operation has properties called *laws of exponents*:

- $b^n b^m = b^{n+m}$
- $(b^n)^m = b^{nm}$
- $(bc)^n = b^n c^n$

Other facts:

- $b^{1/n} = \sqrt[n]{b}$ (defined only for $b > 0$)
- $b^{-n} = \frac{1}{b^n}$

Definition: The inverse of b^x is $\log_b y$: If $b^x = y$ then $log_b y = x$. The number b is called the *base* of the logarithm. The expression $log_b y$ is defined only for b a natural number and y a real number greater than 0.

We will be most interested in base 2.

The logarithm operation has the following properties:

- $\log_b(xy) = \log_b x + \log_b y$
- $\log_b(x/y) = \log_b x - \log_b y$
- $\log_b x^c = c \log_b x$
- $b^{\log_b y} = y$
- $\log_a x = \frac{\log_b x}{\log_b a}$

The last rule (above) is used for *changing bases*.