
CS 380

Home work 1: Solution

If you find any error in the solution (Email to rnath1@cs.utk.edu)

(3 points) Problem 1(a): $\sum_{i=0}^{n} i = n(n+1)/2$

Basis: For n=0;

L.H.S=0

R.H.S=0

L.H.S=R.H.S

Let for n=m, $\sum_{i=0}^{m} i = \frac{m(m+1)}{2}$

Now we have to prove it for n=m+1

L.H.S=$\sum_{i=0}^{m+1} i = \sum_{i=0}^{m} i + (m+1) = \frac{m(m+1)}{2} + (m+1) = \frac{(m+1)(m+2)}{2}$

R.H.S = $\frac{(m+1)(m+2)}{2}$

L.H.S=R.H.S

(3 points) Problem 1(b): $\sum_{i=0}^{n} i^3 = (\sum_{i=0}^{n} i)^2$

Basis: For n=0

L.H.S = $\sum_{i=0}^{n} i^3 = 0$

R.H.S = $(\sum_{i=0}^{n} i)^2 = 0$

Let for n =m, $\sum_{i=0}^{m} i^3 = (\sum_{i=0}^{m} i)^2$

We have to prove it for n=m+1

L.H.S=$\sum_{i=0}^{m+1} i^3 = (m+1)^3 + \sum_{i=0}^{m} i^3 = (m+1)^3 + (\sum_{i=0}^{m} i)^2 = (m+1)^3 + \frac{m^2(m+1)^2}{4} = \frac{(m+2)^2(m+1)^2}{4}$

R.H.S=$(\sum_{i=0}^{m+1} i)^2 = \frac{(m+2)^2(m+1)^2}{4}$

L.H.S=R.H.S

(3 points) Problem 2: Use modular arithmetic to prove that there exists an integer *N* such that for every $n \geq N$ there are integers *a* and *b* for which $n = 3a + 7b$.

Every integer number can be represented as 3m, 3m+1 or 3m+2 for some integer m.

For the numbers like 3m we are done it's already in the format 3a+7b for a=m and b=0.
For numbers like 3m+1, 3m+1=3m+1+6-6=3m-6+7=3(m-2)+7. Now it's in the 3a+7b format with a=m-2 and b=1.
For numbers like 3m+2, 3m+2=3m+2+7-7=3m+9-7=3(m+3)-7. Now it's in the 3a+7b format with a=m+3 and b=-1.

So we are seeing that every integer can be represented in 3a+7b for some integer a and b.
Whatever we choose for integer N, for every n ≥ N, there are integers a and b for which n = 3a + 7b.

(9 points) Problem 3: Let S denote some fixed finite set, and let $P(S)$ denote the power set of S. Define the relation R over $P(S)$ by ARB iff A and B have the same cardinality.

(a) Define a subrelation of R that is reflexive and symmetric but not transitive.
(b) Define a subrelation of R that is reflexive and transitive but not symmetric
(c) Define a subrelation of R that is symmetric and transitive but not reflexive.

Let S be some fixed finite set of anything with at least 3elements.

Define any total ordering among the members of the set S satisfying:

1. For all $a, b \in S$, exactly one of $a < b, a > b, a = b$ is true.
2. If $a < b$ and $b < c$ then $a < c$

The ordering doesn't matter as long as it is a total order. We can take any permutation of the members of set S and define it as our total order.

If S is a set of numbers then this order can be defined by the magnitude of the number.
If S is a set of color then we can define the order with the intensity of the colors.
If S is a set of fruits then we can define the order with the sweetness of the fruit.

P(S) is the power set of S.

R={<A,B>: |A|=|B|}

(a) $R' = \{< A, B >: < A, B > \in R \wedge (A \cap B \neq \emptyset)\} \cup \{< \emptyset, \emptyset >\}$

(b) $R' \quad = \{< A, B >: < A, B > \in R \wedge (A \leq B)\}$

Let,

A=$\{a_1, a_{2,\dots\dots\dots,}\ a_{|A|}\}$

B=$\{b_1, b_{2,\dots\dots\dots,}\ b_{|B|}\}$

After sorting A according to the total order defined earlier we will get a |A|-tuple.

$$< a'_1, a'_{2,\dots\dots\dots,}\ a'_{|A|} > \ where\ \forall i: 1 \leq i < |A|: a'_i < a'_{i+1}$$

For B we will get,

$$< b'_1, b'_{2,\dots\dots\dots,}\ b'_{|B|} > where\ \forall i: 1 \leq i < |B|: b'_i < b'_{i+1}$$

Define $A \leq B\ iff\ (|A| = |B|\ ) \wedge (\forall i: 1 \leq i \leq |A|: a'_i \leq b'_i)$

(c) $R' = \{< A, B >: < A, B > \in R\} / \{< \emptyset, \emptyset >\}$

Note that: We have restricted the set S to have at least 3 members to avoid some special cases e.g. for set with 0 elements, there is no solution for (a).