

CS 380 Theory of Computation  
Spring 2008  
Homework 1  
Due Jan 17

1. Use induction to prove

$$(a) \sum_{i=0}^n i = \frac{n(n+1)}{2}$$

$$(b) \sum_{i=0}^n i^3 = \left( \sum_{i=0}^n i \right)^2$$

2. Use modular arithmetic to prove that there exists an integer  $N$  such that for every  $n \geq N$  there are integers  $a$  and  $b$  for which  $n = 3a + 7b$ .

3. Let  $S$  denote some fixed finite set, and let  $P(S)$  denote the power set of  $S$ . Define the relation  $R$  over  $P(S)$  by  $ARB$  iff  $A$  and  $B$  have the same cardinality.

(a) Define a subrelation of  $R$  that is reflexive and symmetric but not transitive.

(b) Define a subrelation of  $R$  that is reflexive and transitive but not symmetric

(c) Define a subrelation of  $R$  that is symmetric and transitive but not reflexive.