

CS 594 - Spring 2005
Understanding Parallel Computing
Assignment 3
February 1, 2006
Due: February 15, 2006

The accuracy of a floating point system can be characterized by a quantity variously known as the *unit roundoff*, *machine precision*, *machine epsilon*, or *macheps*. Its value, which we denote by ϵ_{mach} , depends on the particular rounding rules used. With rounding by chopping,

$$\epsilon_{mach} = \beta^{1-p},$$

(where β is the base and p is the number of digits in the mantissa) whereas with rounding to nearest

$$\epsilon_{mach} = 1/2 \beta^{1-p}.$$

The unit roundoff is important because it determines the maximum possible relative error in representing a nonzero real number x in a floating point system.

A characterization of the unit roundoff that you may sometimes see is that it is the smallest number ϵ such that

$$fl(1 + \epsilon) > 1.$$

Here $fl(x)$ is the floating point approximation to x .

Part 1:

Compute the machine precision on your computer. In addition compute the overflow and underflow threshold. Explain how you did the computation and your results.

Part 2:

Explain why an alternating infinite series, such as

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

for $x < 0$, is difficult to evaluate accurately in floating point arithmetic.

Part 3:

What happens when you evaluate the infinite series?

$$\sum_{n=1}^{\infty} \frac{1}{n}$$

Explain why summing the series in floating point arithmetic yields a finite sum.