On the application of linear equality constraints in distributed memory

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Abstract

A linear system $Ku = b$ or eigensystem $KX = MX\Lambda$ can be subject to a set of linear constraint equations $Cu = f$. The idea is to use the constraint equations to eliminate a portion of the variables, leaving a reduced linear system $\widehat{K}_{\mathcal{I},\mathcal{I}} u_{\mathcal{I}} = \widehat{b}_{\mathcal{I}}$ or reduced eigensystem $\widehat{K}_{\mathcal{I},\mathcal{I}} X_{\mathcal{I},\mathcal{I}} = \widehat{M}_{\mathcal{I},\mathcal{I}} X_{\mathcal{I},\mathcal{I}} \Lambda_{\mathcal{I},\mathcal{I}}$ defined on independent variables $\mathcal{I}$.

Once the solution vector $u_{\mathcal{I}}$ or eigenvector $x_{\mathcal{I}}$ has been computed, the values at the dependent variables $\mathcal{D}$ (eliminated using the constraint equations) can be recovered via $u_{\mathcal{D}} = \widehat{f}_{\mathcal{D}} - \widehat{C}_{\mathcal{D},\mathcal{I}} u_{\mathcal{I}}$ or $x_{\mathcal{D}} = -\widehat{C}_{\mathcal{D},\mathcal{I}} x_{\mathcal{I}}$.

The above computations require sparse-matrix times sparse-matrix, and sparse-matrix times sparse-vector operations. In a distributed environment, all matrices and vectors are distributed across processors. Their entries needed to be gathered and summed and distributed in some reasonable manner. The above computations must take place, again distributed across the processors. Our approach replaces the relatively simple matrix-matrix and matrix-vector multiplies by sequences of serial computations followed by collective "all-sum" and "all-gather" operations. We will discuss variants of these important collective operations.

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