High performance computation of 3D integral operators on distributed memory computers

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In this work we discuss the solution of 3D integral operators by FEM on the rectangular non-regular grids and its high performance implementation on distributed memory computers. This problem leads to the dense multilevel matrix and can be treated by algorithm described in [1], so we should make the following three steps:

- matrix element calculation and approximation of the matrix by the sum of multilevel Kronecker product of matrices;
- generate a good preconditioner for this matrix;
- solve the preconditioned linear equation system by GMRES method.

The first and the second steps are based on the factor decomposition and the last step need a parallel matrix-vector and preconditioner-vector multiplications and the parallel GMRES. Since the problem is structured, then the fastest possible implementation of GMRES is to distribute each subspace vector regularly through all processors. Thus the main difficulty is on the high performance parallelization of factor decomposition.

The last one we can outline in the following way, suppose $\{a_{ijk}\}_{ijk=1}^{n_1 n_2 n_3}$ is given, and we are searching for the factor decomposition:

$$\min_{B,C,D,E} \sum_{i,j,k} (a_{ijk} - \sum_{l}^{r} \alpha_l b_{il} c_{jl} d_{kl})^2,$$

$$\begin{array}{lll} B = \{b_{il}\} & i = 1, \ldots, m_1 & \\ C = \{c_{jl}\} & j = 1, \ldots, m_2 & l = 1, \ldots, r \\ D = \{d_{kl}\} & k = 1, \ldots, m_3 & \end{array}$$

$$E = \mathrm{diag}(\alpha_1, \ldots, \alpha_r), \quad ||b_l||_2 = ||c_l||_2 = ||d_l||_2 = 1,$$

The parallel algorithm for this problem was developed and tested on the 49 PE Xeon 2.4 GHz cluster with the total main memory of 98 GB. The huge dense linear system with 10^9 unknowns was solved in several hours, the performance was up to 56 GFlop/s.

References

[1] I. Ibraghimov. Application of the three-way decomposition for matrix compression. Num. Lin. Alg. and Appl. 2002, (9), 551-565.

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