Reliable Computation of Equilibrium States and Bifurcations in Nonlinear Dynamics

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Abstract

A problem of frequent interest in the analysis of nonlinear ODE systems is the location of equilibrium states and bifurcations. Through the use of bifurcation diagrams, a large amount of information concerning the number and stability of equilibria in an ODE model can be concisely represented. Bifurcations of equilibria are typically found by solving a nonlinear algebraic system consisting of the equilibrium (steady-state) conditions along with one or more augmenting functions [1]. Typically this equation system is solved using some continuation-based tool (e.g., AUTO). However, in general, these methods do not provide any guarantee that all bifurcations will be found, and are often initialization sensitive. Thus, without some a priori knowledge of system behavior, one may not know with complete certainty if all bifurcation curves have been identified and explored. In this presentation, we explore the use of interval-Newton techniques for identifying, with certainty, *all* equilibrium states and *all* codimension-1 and codimension-2 bifurcations of interest within specified model parameter intervals.

A problem of particular interest is the identification of equilibrium states and bifurcations in food chain and food web models. These models are descriptive of a wide range of behavior in the environment, and are potentially useful in applications such as environmental risk assessment (e.g., [2]). While these models often appear to be mathematically simple, they frequently exhibit a rich mathematical behavior in how the number and stability of equilibria vary with changes in the model parameters (e.g., [3]). Thus, bifurcation analysis is quite useful when characterizing ecological system behavior as it allows for the concise study of a wide range of model parameter values. In this presentation, we will focus on the solution of tritrophic food chains in a chemostat (e.g., Canale's model), applying an interval methodology to ensure that all equilibrium states and bifurcations of interest are identified.

References

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